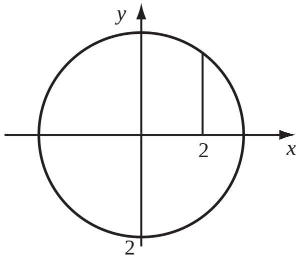
Worked Solutions

Edexcel C4 Paper D

1. (*a*)



(2)

(*b*) volume $= \pi \int_0^2 y^2 \; dx = \pi \int_0^2 (9 - x^2) dx$

$$= \pi \left[9x - \frac{1}{3}x^3\right]_0^2 = \pi \left(18 - \frac{8}{3} - 0\right)$$

$$= \frac{46}{3}\pi \text{ cubic units.}$$ **(4)**

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2. (*a*) $\cos^3 x = \cos x \cdot \cos^2 x$

$= \cos x(1 - \sin^2 x) = \cos x - \cos x \sin^2 x$ **(1)**

(*b*) $\frac{d}{dx}(\sin x)^3 = 3(\sin x)^2 \cdot \cos x$ **(1)**

(*c*) $\int \left(\cos x - \cos x \sin^2 x\right) \; dx = \sin x - \frac{1}{3}\sin^3 x + c$ **(3)**

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3. Given $\frac{dS}{dt} = 640$ cm^2 s^{-1}. To find $\frac{dr}{dt}$.

$S = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

when $r = 5$, $\quad 640 = 8\pi \times 5 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{640}{40\pi}$

$= \frac{16}{\pi}$ cm s^{-1} **(4)**

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4. (*a*) $2x + 2y\frac{dy}{dx} - 2 + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y + 4) = 2 - 2x$

$\frac{dy}{dx} = \frac{2 - 2x}{2y + 4}$

$= \frac{1 - x}{y + 2}$ **(2)**

(*b*) at (4, 2) gradient of tangent $= \frac{1 - 4}{2 + 2} = -\frac{3}{4}$

equation of tangent is $y - 2 = -\frac{3}{4}(x - 4)$

$4y + 3x = 20$ **(4)**

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5. (*a*) $x = \frac{\ln 11}{\ln 5} \left(\text{or } \frac{\log 11}{\log 5}\right) = 1.49$ **(2)**

(*b*) $y = 3^x$

$\ln y = \ln 3^x$

$\ln y = x \ln 3$

$\frac{1}{y}\frac{dy}{dx} = \ln 3$

$\frac{dy}{dx} = y \ln 3 = 3^x \ln 3$ **(3)**

(*c*) $e^{-0.4x} = 5$

$\ln e^{-0.4x} = \ln 5$

$-0.4x = \ln 5$

$x = -\frac{\ln 5}{0.4} = -4.02$ **(2)**

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6. (*a*) $y = x + 9x^{-1}$

$\frac{dy}{dx} = 1 - \frac{9}{x^2}$

at min. point $\frac{9}{x^2} = 1, \quad x = 3 \qquad (x > 0)$

when $x = 3,\ y = 3 + \frac{9}{3} = 6$

so $y \geq 6.$ **(4)**

(*b*) area $= \int_3^9 \left(x + \frac{9}{x}\right) dx = \left[\frac{x^2}{2} + 9\ln x\right]_3^9 = \frac{81}{2} + 9\ln 9 - \left(\frac{9}{2} + 9\ln 3\right)$

$= 36 + 9\ln 3.$ **(4)**

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7. (*a*) $\frac{dy}{dt} = 4e^{4t} - 3, \qquad \frac{dx}{dt} = 4e^{2t} - 1$

$\frac{dy}{dx} = \frac{4e^{4t} - 3}{4e^{2t} - 1}$ **(3)**

(*b*) gradient $= 3, \qquad \frac{4e^{4t} - 3}{4e^{2t} - 1} = 3$

$4e^{4t} - 3 = 12e^{2t} - 3$

$e^{4t} = 3e^{2t}$

$e^{2t} = 3$

$2t = \ln 3$

$t = \frac{1}{2}\ln 3$ **(5)**

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8. (*a*) $P : 2\mathbf{i} + \mathbf{k}, \quad Q : \mathbf{j} + 2\mathbf{k}$ **(2)**

(*b*) $\overrightarrow{QP} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$. equation of line QP is $r = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ **(2)**

(*c*) let $\angle OPQ = \theta$

$\overrightarrow{PO} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}, \overrightarrow{PQ} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, |\overrightarrow{PO}| = \sqrt{5}, |\overrightarrow{PQ}| = \sqrt{6}$

$\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \sqrt{5}\sqrt{6}\cos\theta$

$3 = \sqrt{5}\sqrt{6}\cos\theta$

$\theta = 56.8°$ **(3)**

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9. (*a*) (i) $\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1 + \sin 2\theta$ **(2)**

(ii) $$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1+\sin 2\theta)\ \mathrm{d}\theta = \left[\theta - \frac{1}{2}\cos 2\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - \frac{1}{2}\cos\pi - \left(\frac{\pi}{4} - \frac{1}{2}\cos\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2} - \frac{1}{2}(-1) - \frac{\pi}{4} + 0 = \frac{\pi}{4} - \frac{1}{2}$$ **(3)**

(*b*) $$\int_0^1 (2x+1)^4 \mathrm{d}x = \left[\frac{1}{5}\cdot\frac{1}{2}(2x+1)^5\right]_0^1 = \frac{1}{10}(3^5-1) = 24.2$$ **(4)**

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10. (*a*) $$\frac{3+5x-x^2}{(2-x)(1+x)^2} \equiv \frac{A(1+x)^2 + B(2-x)(1+x) + C(2-x)}{(2-x)(1+x)^2}$$

$3+5x-x^2 \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$

let $x=-1$, $3-5-1 = C(2-(-1)), C=-1$

let $x=2$, $3+10-4 = A(1+2)^2, A=1$

constants, $3 = A + 2B + 2C, B = 2$ **(4)**

(*b*) $$\int_0^1 \left(\frac{1}{2-x} + \frac{2}{1+x} - \frac{1}{(1+x)^2}\right)\mathrm{d}x$$

$$= \left[-\ln(2-x) + 2\ln(1+x) + \frac{1}{(1+x)}\right]_0^1$$

$$= -\ln 1 + 2\ln 2 + \frac{1}{2} - (-\ln 2 + 2\ln 1 + 1) = 3\ln 2 - \frac{1}{2}$$ **(6)**

(*c*) $$\frac{1}{2-x} = \frac{1}{2\left(1-\frac{x}{2}\right)} = \frac{1}{2}\left(1-\frac{x}{2}\right)^{-1}$$

$$= \frac{1}{2}\left[1 + (-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-2)}{2}\left(-\frac{x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{3.2}\left(-\frac{x}{2}\right)^3 + \ldots\right]$$

$$= \frac{1}{2}\left[1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8}\right]$$

$$= \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16}$$

$$\mathrm{f}(x) = \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16} + 2(1+x)^{-1} - (1+x)^{-2}$$

$$= \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16}$$

$$+2\left[1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3.2}x^3 + \ldots\right]$$

$$-\left[1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3.2}x^3 + \ldots\right]$$

$$= \left(\frac{1}{2} + 2 - 1\right) + x\left(\frac{1}{4} - 2 + 2\right) + x^2\left(\frac{1}{8} + 2 - 3\right) + x^3\left(\frac{1}{16} - 2 + 4\right)$$

$$= \frac{3}{2} + \frac{1}{4}x - \frac{7}{8}x^2 + \frac{33}{16}x^3.$$ **(4)**

(*d*) valid for $-1 < x < 1$. **(1)**
